where the velocity of the impacting plates U is directed along the normal to their surfaces, this condition is written in the form

$$\tan \alpha = U/c_0.$$

Making use of the fundamental relation of the hydrodynamic theory of jetting, relating the coherent jet velocity to the impact parameters, we obtain

$$v_j = \frac{U}{\tan \alpha}\left(1 + \sqrt{1 + \tan^2 \alpha}\right) = c_0 + \sqrt{c_0^2 + U^2}.$$

Thus, the velocity of a solid coherent jet is more than twice the sound velocity of the jet material. In principle, therefore, a gun that uses a coherent jet as a piston is capable of accelerating firing pins to more than four times the sound velocity in the jet material under standard conditions.

It is essential to note that this hybrid of a light-gas gun and explosive accelerators consolidates the advantages of both driving techniques. In contrast with the gas-jetting shaped charges used to accelerate rigid bodies [7], the weight of the HE charge in the coherent-jet gun is an order of magnitude smaller.

LITERATURE CITED

1. M. A. Lavrent'ev, "The shaped charge and its principle of operation," Usp. Mat. Nauk, 12, No. 4(76) (1957).
2. G. Birkhoff, D. P. MacDougall, E. Puch, and G. Taylor, "Explosives with lined cavities," J. Appl. Phys., 19 (1948).
3. V. M. Titov, "Possible hydrodynamic jetting regimes in the collapse of a liner," Dokl. Akad. Nauk SSSR, 247, No. 5 (1979).
4. R. J. Eichelberger and J. Kineke, "Hypervelocity impact," in: Kurzzeitphysik (K. Vollrath and G. Thomer, eds.), Springer-Verlag, Vienna (1967), pp. 659-692.
5. C. L. Lecomte, "Lancement de maquettes à grande vitesse," in: Kurzzeitphysik (K. Vollrath and G. Thomer, eds.), Springer-Verlag, Vienna (1967), pp. 693-718.
6. P. C. Chou, J. Carleone, and R. R. Karpp, "Criteria for jet formation from impinging shells and plates," J. Appl. Phys., 47, No. 7 (1976).
7. V. M. Titov and G. A. Shvetsov, "Laboratory methods for the explosive high-velocity launching of rigid bodies," Fiz. Goreniya Vzryva, No. 3 (1970).

DISTRIBUTION OF TIMES TO FRACTURE UNDER RANDOM LOADING

V. V. Bolotin

UDC 539.2:539.4

Predicting the times for a structural element to attain a certain hazardous state (such as fracture) is important both from the standpoint of new structural designs and from the standpoint of monitoring the instantaneous state of structures in service. In the latter case the prediction results are used to solve the problem of the advisability or safety of continued service of the particular structure, necessary preventive measures, etc. From the vantage point of mechanics, time-to-fracture prediction poses a complex problem, which includes describing the defect accumulation process and the development of macroscopic cracks in the structure, as well as estimating the loss of bearing capacity of a defective structure and its life expectancy under the conditions of loading, which is generally of a random nature and is specified by certain a priori distribution functions. In this article we develop a defect-accumulation model for structural elements, which is conditionally separable into two stages: 1) incubation; 2) propagation of arterial cracks. In this connection a relationship is postulated between a phenomenological measure of the defective state, which depends on the loading process, and the expectation value of the number of macroscopic cracks nucleating in a certain reference volume. Another significant aspect of the approach developed here is the application of the central limit theorem for asymptotic estimation of the distribution functions of nonstationary random processes to characterize the accumulation of defects in the structural element and its residual bearing capacity.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 149-158, September-October, 1980. Original article submitted March 26, 1980.

0021-8944/80/2105-0699$07.50

1. We consider a body of volume V, which is subjected to the prolonged action of random slowly varying or cyclic loads. We assume in regard to the latter that their parameters vary slowly enough for the cyclic loading process to be regarded approximately as proceeding continuously in "slow" time t. We describe the nominal stress field in the body by means of the vector function $\mathbf{s}(\mathbf{x}, t)$, where $\mathbf{x}$ is the radius vector of points of the body. We partition the body into sections, each with a volume of the order V_0. We choose these sections so that the space scale of each will be sufficiently large in comparison with the structural scale of the material and with the characteristic length l_* of a nucleating macroscopic crack. On the other hand, the dimensions of these sections must be sufficiently small in comparison with the characteristic scales of variation of the macroscopic stresses and macroscopic properties of the volume with respect to the volume V_0. In the volume V_0 we consider the field $\mathbf{s}(\mathbf{x}, t)$ to be independent of x and denote it simply by $\mathbf{s}(t)$. For simplification we also assume that the characteristic length l of the developed macroscopic crack does not exceed the limits of the volume V_0 before this length attains the maximum admissible value or the critical value l_{**}. We refer to V_0 hereinafter as the reference volume. Some of these restrictions are introduced merely to simplify the expressions and can be lifted without particular difficulty.

We consider the first (incubation) stage, during which nuclei of macroscopic cracks with characteristic lengths l_* are formed in the weakest or maximally stressed structural elements. We characterize the degree of readiness of the material to form a nucleus in the reference volume V_0 by the scalar defect measure ψ. This measure is a nondecreasing function of t and at each instant is a functional of the load history in the interval [0, t]. Inasmuch as $\mathbf{s}(t)$ is a random process, $\psi(t)$ will be likewise. The distribution of its values at each instant is characterized by the distribution function $F_\psi(\psi; t)$ and the corresponding probability density function $p_\psi(\psi; t)$. The methods for calculating these characteristics for a given relation between the defect measure and the loading process in general are well known [1]; certain details will be discussed below.

We denote the number of macroscopic cracks or crack nuclei in the reference volume V_0 by k. This quantity is an integer-valued random process k(t). The experimeter declares the end of the incubation stage upon observing the first macroscopic crack in the sample. It is therefore logical to postulate the relationship between the defect measure $\psi(t)$ and the expectation $\nu(t) = \mathbf{E}[k(t)]$ of the number of macroscopic cracks or nuclei. The expectation $\mathbf{E}(\cdot)$ is evaluated with respect to the statistical ensemble of analogous volumes V_0 determined under statistically uniform conditions. We thus assume that

$$\nu = f(\psi),\ 0 \leqslant \psi < \infty, \tag{1.1}$$

where $f(\psi)$ is a continuously differentiable function staisfying the conditions $f'(\psi) > 0$, $f(0) = 0$, $f(1) = 1$. Unlike the conventional treatment, here we continue the defect measure analytically on the interval $(1, \infty)$. It is evident from these considerations that the reference volume must be of the order of the volume of the standard samples used in cyclic or long-time strength testing.

The formation of nuclei of macroscopic cracks is an infrequent event. The characteristic length of the nuclei is such that with probability close to unity it is permissible to neglect their mutual influence and to adopt a Poisson distribution for their number k. Then the probability that exactly k macroscopic cracks or nuclei will occur in V_0, subject to the condition that the defect measure is equal to a prescribed value of ψ, is defined as

$$Q_k = \frac{f^k(\psi)}{k!} \exp[-f(\psi)], \quad k = 0, 1, \ldots \tag{1.2}$$

Analogously, for the probability of the occurrence of at least one crack or nucleus in V_0 we have the expression

$$Q(\psi) = 1 - \exp[-f(\psi)]. \tag{1.3}$$

Here the functions $Q_k(\psi)$ and $Q(\psi)$ have the significance of conditional probabilities. If the distribution function $F_\psi(\psi; t)$ of the measure ψ, $0 \leqslant t < \infty$, has been found for a given loading process $\mathbf{s}(t)$, then the probability can be calculated for the event that at least one macroscopic crack occurs in the volume V_0 at time t:

$$Q(t) = \int_0^\infty \{1 - \exp[-f(\psi)]\}\, dF_\psi(\psi; t). \tag{1.4}$$

The time t_* of inception of the first crack is logically treated as the end of the incubation state. Thus, inasmuch as the stress state in V_0 is uniform by assumption, the first-formed crack has a preferential chance for further growth and, as a rule, becomes the cause of fracture. Here we intentionally refrain from the complications introduced by crack interaction, branching, etc. If we take this point of view, then expression (1.4) yields the distribution function of the termination times of the incubation stage:

$$F_*(t_*) = \int_0^\infty \{1 - \exp[-f(\psi)]\}\, dF_\psi(\psi; t_*). \quad (1.5)$$

The macroscopic crack growth stage is described by the equations of fracture mechanics [2]. If the loading process $\mathbf{s}(t)$ is random, then the results of solving these equations will be the formulation of the distribution function $F_l(l; t|t_*)$ of the lengths l of cracks nucleating at time t_*. In turn, t_* is a random variable with the distribution function (1.5). Accordingly, for the unconditional distribution of the length l of the first-nucleated crack we obtain the expression

$$F_l(l; t) = \int_0^t F_l(l; t\,|\,t_*)\, dF_*(t_*). \quad (1.6)$$

Let the critical (or limiting) crack length be independent of $\mathbf{s}$ and equal to a prescribed value l_{**}. Then the time t_{**} for a crack nucleating at time t_* to attain the critical length l_{**} has the conditional distribution function

$$F_{**}(t_{**}\,|\,t_*) = 1 - F_l(l_{**}; t_{**}\,|\,t_*). \quad (1.7)$$

The unconditional distribution function is defined by the equation

$$F_{**}(t_{**}) = \int_0^{t_{**}} F_{**}(t_{**}\,|\,t_*)\, dF_*(t_*). \quad (1.8)$$

The critical length l_{**} usually depends on the nominal stress level at the leading edge of the crack at a given time. This fact somewhat complicates the problem of determining the distribution function for t_{**} insofar as it necessitates the simultaneous analysis of two stochastically related processes $l(t)$ and $l_{**}(t)$. The time t_{**} is found from the condition of intersection of these two processes.

2. The foregoing considerations are of a general character. We now indicate a class of problems for which an approximate (in the asymptotic sense) solution is more or less readily obtainable. Let the defect-accumulation process be described by the differential equation

$$d\psi/dt = f_1(\psi) f_2(\mathbf{s}), \quad (2.1)$$

in which $f_1(\psi)$ and $f_2(\mathbf{s})$ are continuous functions of the defect measure ψ and the stress vector $\mathbf{s}$, respectively. Here $f_1(\psi) > 0$, $f_2(\mathbf{s}) \geqslant 0$. The initial conditions for Eq. (2.1) are taken in the form $\psi(0) = 0$ for the nondefective material and in the form $\psi(0) = \psi_0$ if the material has a defect ψ_0 at time $t = 0$. Let the growth of the characteristic crack length l be described by the analogously structured equation

$$dl/dt = g_1(l) g_2(\mathbf{s}), \quad (2.2)$$

in which $g_1(l) > 0$ and $g_2(\mathbf{s}) \geqslant 0$ are continuous functions of the crack length l and the stress vector $\mathbf{s}$, respectively. The initial condition for Eq. (2.2) is the nucleation of a macroscopic crack at time t_*: $l(t_*) = l_*$. The process $l(t)$ is defined for $t_* \leqslant t < \infty$. However, since the crack growth is limited by considerations of the reliability of the limiting value l_{**}, the upper limit of the interval is taken as t_{**}, where $l(t_{**}) = l_{**}$. In this section we regard the quantity l_{**} as given; the case in which l_{**} depends on the initial stress vector $\mathbf{s}(t)$ at a given time t will be discussed below.

The form of Eqs. (2.1) and (2.2) is chosen so that the variables in them will be separable. The majority of phenonenological equations describing the accumulation of defects in long-term or cyclic loading [1] fit into the scheme of (2.1). Among the equations of fracture mechanics, a typical representative of the type (2.2) is the Paris–Erdogan equation [2], which describes the subcritical growth of cracks in cyclic loading:

$$\frac{dl}{dt} = \frac{c}{\tau_0}(\Delta K)^n, \quad (2.3)$$

where c and n are certain positive empirical constants, τ_0 is the time constant, and ΔK is the range of variation of the stress intensity factor K. In turn, $K = \gamma s\sqrt{l}$, where s is the nominal stress at the leading edge of the crack and γ is a coefficient of order unity. On the other hand, the equations in [3], which take into account the threshold value of the intensity factor, are not of the type (2.2).

Reducing Eqs. (2.1) and (2.2) to quadratures and invoking the initial conditions, we obtain

$$U(\psi) = u(t),\ W(l) = w(t|t_*). \tag{2.4}$$

On the left-hand sides of relations (2.4) are the functions

$$U(\psi) = \int_0^\psi \frac{d\psi}{f_1(\psi)},\quad W(l) = \int_{l_*}^l \frac{dl}{g_1(l)}, \tag{2.5}$$

and on the right-hand sides the functions

$$u(t) = \int_0^t f_2[s(\tau)]\,d\tau,\quad w(t\,|\,t_*) = \int_{t_*}^t g_2[s(\tau)]\,d\tau. \tag{2.6}$$

With the constraints on the functions entering into Eqs. (2.1) and (2.2), all the integrals in (2.4) and (2.5) exist. The domains of variation of the left- and right-hand sides of Eqs. (2.4), generally speaking, do not coincide, a fact that must be taken into consideration in the ensuing calculations.

Inasmuch as $s(t)$ is a random process, the functions $u(t)$ and $w(t|t_*)$ determined according to (2.6) will be likewise. For a certain set of conditions imposed on the integrands in (2.6) the central limit theorem for integrals of random processes extends to the functions $u(t)$ and $w(t|t_*)$. The conditions of this theorem, which is rigorously formulated in [4], include not only the usual constraints on the moments of the integrands, but also the requirements of sufficient mixing of the processes. In turn, the sufficient mixing condition can be reduced to the requirement that the characteristic correlation time τ_c of the integrands be sufficiently small in comparison with the time interval in which the integration is carried out. It can be assumed without essential limitations that τ_c is of the order of the characteristic correlation time of the loading process $s(t)$. Since the central limit theorem is used mainly to estimate the probabilities referred to the times t_* and t_{**}, the mixing requirement is logically written in the form

$$\tau_c \ll \min\{t_*,\ t_{**} - t_*\}. \tag{2.7}$$

Let the conditions of the central limit theorem from [4] be satisfied. Then for the distribution function $F_\psi(\psi;\ t)$ of the defect measure ψ at time t we have the asymptotic representation

$$F_\psi(\psi;\ t) \sim \Phi\left\{\frac{U(\psi) - \mathbf{E}[u(t)]}{\sqrt{\mathbf{D}[u(t)]}}\right\}, \tag{2.8}$$

in which $\mathbf{D}(\bullet)$ is the variance and $\Phi(u)$ is the standard normal distribution function

$$\Phi(u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^u \exp\left(-\frac{1}{2}z^2\right) dz.$$

The expectation $\mathbf{E}[u(t)]$ and the variance $\mathbf{D}[u(t)]$ of the auxiliary process $u(t)$ are calculated as

$$\mathbf{E}[u(t)] = \int_0^t \mathbf{E}\{f_2[s(\tau)]\}\,d\tau,$$
$$\mathbf{D}[u(t)] = \int_0^t\int_0^t \mathbf{E}\{f_2^*[s(\tau_1)]\,f_2^*[s(\tau_2)]\}\,d\tau_1 d\tau_2. \tag{2.9}$$

Here $f_2^*[s(\tau)]$ is the corresponding centered process. We obtain an analogous asymptotic relation for the distribution function $F_l(l;\ t|t_*)$ of the length l of a crack nucleating at time t_*:

$$F_l(l;\ t\,|\,t_*) \sim \Phi\left\{\frac{W(l) - \mathbf{E}[w(t\,|\,t_*)]}{\sqrt{\mathbf{D}[w(t\,|\,t_*)]}}\right\}. \tag{2.10}$$

For a fixed t the right-hand sides of expressions (2.8) and (2.10), generally speaking, do not tend to unity as $\psi \to \infty$ and $l \to \infty$, respectively. This means that for finite t the probability of detecting arbitrarily large values of ψ or l can have a nonzero value. The probabilities of these events are defined as the compliments of the distribution functions $F_\psi(\psi, t)$ and $F_l(l; t|t_*)$ to unity as $\psi \to \infty$ and $l \to \infty$, respectively.

3. As an illustrative example we consider cyclic loading with nominal stresses representing a one-dimensional narrowband normal process with expectation zero, variance σ^2, and characteristic (effective) period τ_0. The amplitudes of this process $s \geqslant 0$ obey a Rayleigh distribution with density function

$$p(s) = \frac{s}{\sigma^2} \exp\left(-\frac{s^2}{2\sigma^2}\right) \tag{3.1}$$

and autocorrelation function $K_s(\tau) = \sigma^2\rho(\tau)$. Here $\rho(\tau)$ is a slowly varying function in a time of order τ_0. For the defect measure ψ we take an equation of the type (2.1):

$$\frac{d\psi}{dt} = \frac{1}{N_c\tau_0}\left(\frac{s}{r}\right)^m, \tag{3.2}$$

where N_c is the base number of cycles, m is the exponent of the fatigue curve, and r is a characteristic stress having the significance of the fatigue limit. For the crack length l we use the Paris—Erdogan equation (2.3). Since the range ΔK of the stress intensity factor is related to the amplitude s and crack length l by the equation $\Delta K = 2\gamma s\sqrt{l}$, expression (2.3) retains the structure of (2.2).

Calculating the left-hand sides of relations (2.4) according to (2.5), we obtain

$$U(\psi) = \psi, \quad W(l) = \frac{1-(l_*/l)^{\frac{1}{2}n-1}}{\frac{1}{2}n-1}. \tag{3.3}$$

The expression for $W(l)$ is suitable for $n \neq 2$. If $n = 2$, then $W(l) = \ln(l/l_*)$. The right-hand sides of relations (2.4), according to (2.6), have the form

$$u(t) = \frac{1}{T_c}\int_0^t \left[\frac{s(\tau)}{r}\right]^m d\tau, \quad w(t|t_*) = \frac{1}{T_{c_1}}\int_{t_*}^t \left[\frac{s(\tau)}{r}\right]^n d\tau, \tag{3.4}$$

where the following notation is introduced for the time constants in order to abridge the formulas:

$$T_c = N_c\tau_0, \quad T_{c_1} = \tau_0\left[c\,(2\gamma r)^n l_*^{\frac{1}{2}n-1}\right]^{-1}. \tag{3.5}$$

The expectations and variances of the processes $u(t)$ and $w(t|t_*)$ are determined from expressions (2.9). In particular, on the basis of (3.1), we obtain for the process $u(t)$ [5]

$$\begin{aligned} \mathbf{E}[u(t)] &= \frac{t}{T_c}\left(\frac{\sigma}{r}\right)^m 2^{\frac{1}{2}m}\,\Gamma\left(1+\frac{1}{2}m\right), \\ \mathbf{D}[u(t)] &\approx \frac{t}{T_c^2}\left(\frac{\sigma}{r}\right)^{2m} 2^m \sum_{j=1}^{\infty} B_{j,m}^2\tau_j. \end{aligned} \tag{3.6}$$

The following notation is used in the second expression (3.6):

$$\begin{aligned} B_{j,m} &= \sum_{k=0}^{j} \frac{(-1)^k\, j!}{(k!)^2\,(j-k)!}\,\Gamma\left(1+k+\frac{1}{2}m\right), \\ \tau_j &= 2\int_0^\infty \rho^{2j}(\tau)\,d\tau, \end{aligned} \tag{3.7}$$

and this expression is approximate, its error diminishing rapidly with increasing value of $t/\tau_c \gg 1$. The convergence of the series in the second expression (3.6) can be proved on the basis of the fact that its coefficients are expressed in terms of the coefficients of the expansion of the two-point density function for a Rayleigh process into a Laguerre polynomial series. For even m the upper limit of the summation in the first expression (2.7) is $j = m/2$.

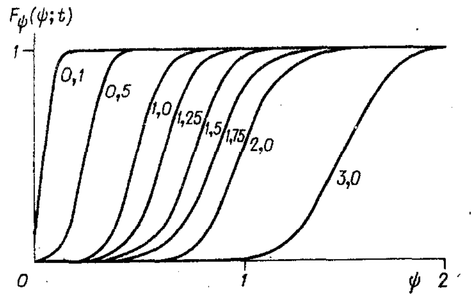

Fig. 1

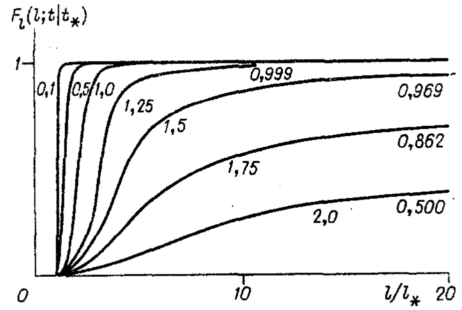

Fig. 2

Analogously, for the process $w(t\ t_*)$ we have

$$\mathbf{E}[w(t\mid t_*)]=\frac{t-t_*}{T_{c_1}}\left(\frac{\sigma}{r}\right)^n 2^{\frac{1}{2}n}\Gamma\left(1+\frac{1}{2}n\right),$$
$$\mathbf{D}[w(t\mid t_*)]\approx\frac{t-t_*}{T^2_{c_1}}\left(\frac{\sigma}{r}\right)^{2n}2^n\sum_{j=1}^{\infty}B^2_{j,n}\tau_j. \tag{3.8}$$

The right-hand side of the second expression, which is more precise the stronger the inequality $(t - t_*)/\tau_c \gg 1$, contains the coefficients $B_{j,n}$, which are calculated from the corresponding expression (3.7) with m replaced by n.

Certain results of calculations based on expressions (1.5)-(1.8), (2.8), (2.10), and (3.6)-(3.8) are given below. The following initial data are used for the examples: $m = n = 4$, $T_c = T_{c_1}$, $\beta = 2$, $\rho(\tau) = \exp(-|\tau|/\tau_c)$, where $\tau_c = 10^{-2}T_c$. Graphs of the distribution function $F_\psi(\psi; t)$ of the defect measure ψ and the distribution function $F_l(l; t|t_*)$ of the length l of a crack nucleating at time t_* are given in Figs. 1 and 2. All the curves are plotted for a dimensionless stress level $\sigma/r = 0.5$ and various relative times t/T_c and $(t - t_*)/T_{c_1}$, respectively. The values of these times are indicated alongside the curves.

For $l \to \infty$ and fixed values of $t - t_*$ the distribution function $F_l(l; t|t_*)$ tends in general to a limit that differs from unity. The complement of this limit to unity is equal to the probability of unbounded growth of the crack for a given $t - t_*$. We note that for $n \leqslant 2$ the domain of variation of the function $W(l)$ is the semiaxis $[0, \infty]$, so that the asymptotic value of the distribution function is equal to unity as $l \to \infty$. It is clear that the behavior of $F_l(l; t|t_*)$ depends strongly on how the rate dl/dt varies with growth of the crack length. This fact is illustrated in Fig. 3, in which the curves of $W(l)$ for $n = 2, 3, 4$ are combined with the curves of the density function $p_w(w; t|t_*)$ for the values of the process $w(t|t_*)$. The hatched region corresponds to the limit of the function $F_l(l; t|t_*)$ for $n = 2$ and $l \to \infty$. On the other hand, since expressions (2.8) and (2.10) are of an asymptotic character, the reliability of the numerical results can suffer appreciably precisely at the "tails" of the distributions.

Figure 4 gives curves of the distribution functions $F_*(t_*)$ and $F_{**}(t_{**}|t_*)$ plotted according to expressions (1.5) and (1.7). The dimensionless times $\tau = t_*/T_c$ and $\tau = (t_{**} - t_*)/T_{c_1}$ are plotted along the respective horizontal axes. The numbers alongside the curves of $F_{**}(t_{**}|t_*)$ indicate the values used for the ratio l_{**}/l_*. The distribution of the times $t_{**} - t_*$ turns out to be much more compact than that of the times t_*. This result is a consequence of the fact that the mechanisms of the accumulation of initial defects and propagation of arterial cracks differ appreciably. Whereas the statistical scatter of the times t_* is generated both by the random distribution of defects in the material and by the random nature of the loading process, the scatter of the times $t_{**} - t_*$ (within the context of the presumed model) is elicited exclusively by the random nature of the loading.

4. We have assumed thus far that the critical or maximum admissible length l_{**} is a given quantity. From the standpoint of linear fracture mechanics the critical crack length depends significantly on the nominal stress level at the leading edge of the crack. We take the nonfracture condition in the form

$$K_{\max}=\gamma s_{\max}\sqrt{l}<K_{**}, \tag{4.1}$$

where $s_{\max}$ is the maximum nominal stress at the leading edge and K_{**} is the critical value of the stress intensity factor. Here the critical length $l_{**}(t)$ is a random time function

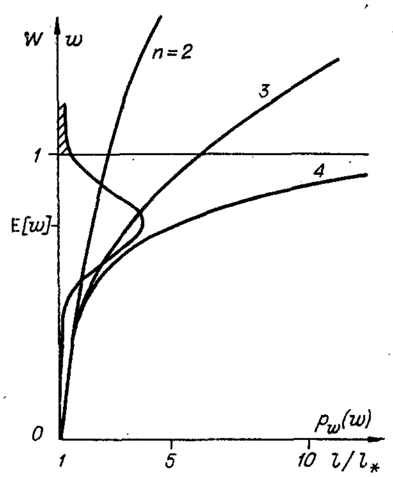

Fig. 3

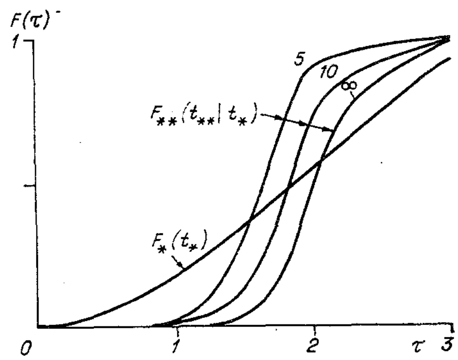

Fig. 4

depending on the loading process s(t). This fact greatly complicates the solution of the problem of determining the distribution functions of the times to fracture, requiring recourse to the methods of the theory of excursions of random processes. This type of problem has been investigated [6] on the assumption that at some initial time t_0 the macroscopic crack length l_0 is given as a deterministic quantity. We now show how to take into account the influence of the random mechanism of the generation of a macroscopic crack on the distribution of times for the crack to attain critical length.

As in [6], we formulate the problem in terms of the nominal stresses, specifying the latter by means of a one-dimensional random process s(t). The corresponding critical stress for a crack nucleating at time t_* is defined, according to (4.1), as

$$s_{**}(t\,|\,t_*) = K_{**}/\gamma\sqrt{l(t\,|\,t_*)}. \tag{4.2}$$

We determine the probability of the event that inequality (4.1) will never fail in the time interval $[t_*, t]$. This probability has the significance of the reliability index of the probability of safe operation in the indicated interval:

$$R(t\,|\,t_*) = \mathbf{P}\{s(\tau) < s_{**}(\tau\,|\,t_*);\ \tau \in [t_*, t]\}. \tag{4.3}$$

The difficulty of calculating this probability stems from the fact that inequality (4.1) must absolutely never fail up to the end of the investigated time interval, when the crack attains its maximum length. As a result of excursions of the process s(t) inequality (4.1) can be violated even in the early stages of crack development. If such excursions are relatively infrequent events, then it is permissible to use the following approximate (under definite conditions, asymptotically exact) relation between the function $R(t|t_*)$ and the expectation $\lambda(t|t_*)$ of the number of excursions of the process s(t) beyond the level $s_{**}(t|t_*)$ per unit time:

$$R(t\,|\,t_*) \approx \exp\left[-\int_{t_*}^{t} \lambda(\tau\,|\,t_*)\,d\tau\right].$$

For the calculation of the expectations of the number of excursions, on the other hand, there are well-developed methods [1, 7].

The conditional distribution function of the times to first violation of inequality (4.1) is defined, obviously, as the complement of the probability (4.3) to unity, and the unconditional distribution function is calculated according to an expression of the type (1.8):

$$F_{**}(t_{**}) = \int_0^{t_{**}} [1 - R(t_{**}\,|\,t_*)]\,dF_*(t_*).$$

To calculate the expectation of the number of violations of the inequality $s(t) < s_{**}(t|t_*)$ per unit time requires knowledge of the joint distribution function of the values of the processes s(t) and $s_{**}(t|t_*)$ as well as their first time derivatives. However, at times of the order $t_{**} - t_* \gg \tau_0$ the process s(t) can be treated as rapidly varying in composition with $s_{**}(t|t_*)$, and within small error limits the stochastic interdependence of the values of these processes at coinciding times can be neglected. These assumptions are better satisfied the stronger the inequality (2.7). Under the stated assumptions the characteristic $\lambda(t|t_*)$ obeys the approximate expression

$$\lambda(t|t_*) \approx \int_0^\infty \int_0^\infty p_1(s_{**}, \dot{s}; t)\, p_2(s_{**}; t|t_*)\, \dot{s} d\dot{s} ds_{**}.$$

Here $v_1(s, \dot{s}; t)$ is the joint density function for the process s(t) and its first derivative $\dot{s}(t)$ at coinciding times, and $p_2(s_{**}; t|t_*)$ is the density function for the process $s_{**}(t|t_*)$ at the same times. The density function $p_1(s, \dot{s}; t)$ is specified as part of the description of the loading process, whereas the distribution of the values of the process $s_{**}(t|t_*)$, according to (4.2), is expressed in terms of the distribution function of the values of the process $l(t|t_*)$.

LITERATURE CITED

1. V. V. Bolotin, Statistical Methods in Structural Mechanics [in Russian], Stroiizdat, Moscow (1965).
2. H. Liebowitz (ed.), Fracture, an Advanced Treatise, Vol. 3, Academic Press, New York (1971).
3. L. M. McCartney and P. M. Cooper, "A new method of analyzing fatigue crack propagation data," Eng. Fracture Mech., 9, No. 2 (1979).
4. I. A. Ibragimov and Yu. V. Linnik, Independent and Stationarily Coupled Variables [in Russian], Nauka, Moscow (1965).
5. V. V. Bolotin, Random Vibrations of Elastic Systems [in Russian], Nauka, Moscow (1979).
6. V. V. Bolotin, "Safe crack dimensions under random loading," Izv. Akad. Nauk SSSR, Mekh. Tverd. Tela, No. 1 (1980).
7. V. I. Tikhonov, Excursions of Random Processes [in Russian], Nauka, Moscow (1970).

CONTINUATION WITH RESPECT TO A PARAMETER IN NONLINEAR ELASTICITY THEORY PROBLEMS

É. I. Grigolyuk and V. I. Shalashilin

UDC 539.3

1. The equations describing the nonlinear static deformation of elastic systems generally contain a parameter, usually the load. We consider algebraic and transcendental equations. The generalization to functional and operator equations presents no difficulties in principle.

Let us consider a system of nonlinear equations for the vector $\mathbf{x} = \{x_1, \ldots, x_m\}$ containing a parameter λ:

$$F(\mathbf{x}, \lambda) = 0, \tag{1.1}$$

where $F = \{F_1(\mathbf{x}, \lambda), \ldots, F_m(\mathbf{x}, \lambda)\}$ is a vector function which is nonlinear with respect to $\mathbf{x}$ and λ, and is assumed continuous and differentiable with respect to $\mathbf{x}$ and λ a sufficient number of times.

Suppose for $\lambda \in [\lambda_0, \lambda_n]$ Eq. (1.1) has the solution $\mathbf{x}(\lambda)$, and that for $\lambda = \lambda_0$ the solution $\mathbf{x}_0 = \mathbf{x}(\lambda_0)$ is known, i.e.,

$$F(\mathbf{x}_{(0)}, \lambda_0) = 0. \tag{1.2}$$

We introduce an (m + 1)-dimensional vector space E_{m+1}: $\{\mathbf{x}, \lambda\}$. In this respect the point corresponding to the solution of (1.1) describes a continuous curve K which passes through the points $\mathbf{x}_{(0)}$, $\lambda_{(0)}$, and $\mathbf{x}_{(n)}$, $\lambda_{(n)}$. The idea of the method of continuation with respect to a parameter consists in constructing a sequence of solutions $\mathbf{x}_{(k)} = \mathbf{x}(\lambda_k)$ (k =

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 158-162, September-October, 1980. Original article submitted March 15, 1980.

0021-8944/80/2105-0706$07.50